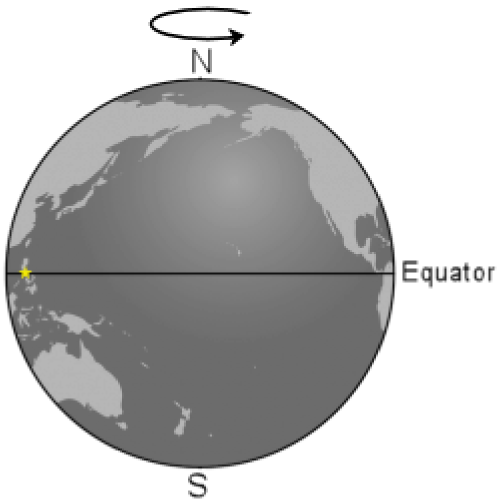
Speed of Earth Rotation at the Equator by Mathematical Modelling

People do not realise that they are actually moving very quickly on the Earth. The reason being that forces such as momentum and gravity impact upon us so that we do not recognise that we are actually moving.

Consider being inside a motor vehicle and driving. The vehicle is moving quite quickly but to a person travelling in the vehicle it appears as though things are almost stationary. A person may not be moving relative to the vehicle but the vehicle is moving relative to its surroundings. The same principle applies to the movement of the Earth.

Consider some point on the Earth's surface on the Equator (imaginary circle that extends around the circumference of the Earth). At the same time, the Earth is spinning on its axis that extends from the North (N) to South (S) poles and which is at right angles to the Equator. This is shown in the diagram below;



Looking at the Earth, suppose there is some point (in this example shown by the yellow star), that is on the Earth's surface and we want to know how fast it is travelling as the Earth rotates.

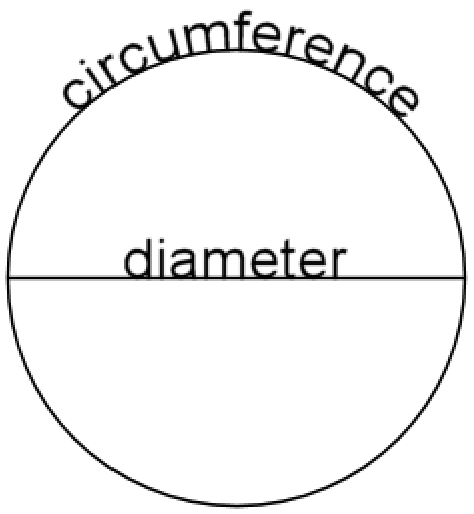
Before the speed can be determined it is essential that all necessary information and relevant Mathematical concepts are known.

Firstly, the Earth is very closely approximated in two dimensions as a circle and so facts associated with a circle can be utilised.

Also, it is important to have an understanding of rates, which is a comparison of two units of measurement that are distinct from each other. For instance, we could have a person who works 40 hours over a 5 day week and so this would happen at the rate of 8 hours per day (8 hours / day). Speed is also a rate that compares the distance travelled over a particular time period with some common units being kilometres per hour (km/h) or metres per second (m/s) or similar.

To determine the speed at which a particular point will be moving on the Earth's surface it is necessary to have the pre-requisite knowledge and to form the appropriate model in order to solve the problem.

The Circle



Viewed as a ‘slice’, the Earth has a circular shape. The larger length around the centre of the Earth at the Equator is its circumference. The diameter of the Earth is the distance from one side to the other through the centre (or core) of the Earth.

For any circle, regardless of size, the length of the diameter divided by the length of the diameter for that circle is always equal to the value 3.14159… which is referred to as pi (π).

This means that;

$$\frac{C}{d} = \pi$$

$$C = \pi d$$

Speed as a Rate

Speed is equivalent to the distance that an object will travel in a given time period. This can be written as;

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$S = \frac{D}{T}$$

Important and Necessary Data

- The diameter (d) of the Earth at the Equator is 12,756.32km.

- The time taken for the Earth to rotate once on it’s North South axis is usually considered to be 24 hours (1 day). There are approximately 365 days in a year but this is not quite correct with the actual figure being 365.25 days in a year. This is the reason why there are 365 days in a year and an extra day every four years (366 days or a Leap Year). One rotation of the Earth happens over a period of 23.93 hours.

Now it simply remains to create a model that can be used to find the rotational speed for a point (person, object, etc) on the surface of the Earth that will turn once in a day.

Solution

Distance Around Equator (Circumference, C)

$$
\begin{aligned}
C &= \pi d \\
&= \pi \times 12{,}756.32 \\
&= 40075.1612 \\
&\approx 40075\text{km}
\end{aligned}
$$

Due to the measurement of the distance around the equator, rounded to the nearest kilometre, the distance of rotation travel by a point at the Equator would be 40,075 km.

Speed (expressed in km/h)

$$
\begin{aligned}
S &= \frac{D}{T} \\
&= \frac{40075km}{23.93h} \\
&= 1674.676139 \\
&\approx 1675\text{km/h}
\end{aligned}
$$

This means that any object on the surface of the Earth lying on the Equator will be travelling in space at a speed of 1,675 km/h.

Speed Conversion (from km/h to metres per second - m/s)

It is important to know the following information in order to convert from one unit of speed (km/h) into another unit of speed (m/s).

$$1km = 1000m$$

$$
\begin{aligned}
1h &= 60\min \\
&= 60 \times 60\sec \\
&= 3600\sec
\end{aligned}
$$

$$
\begin{aligned}
S &= \frac{D}{T} \\
&= \frac{40075 \times 1000\text{m}}{23.93 \times 60 \times 60\text{s}} \\
&= 465.1878163 \\
&\approx 465\text{m/s}
\end{aligned}
$$

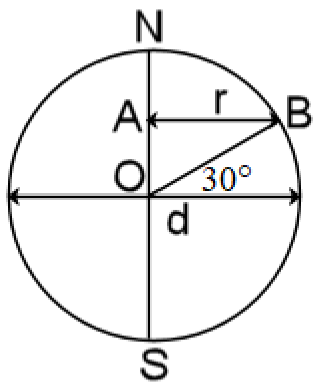
Hence, any object on the Equator would be travelling at about 465 m/s. In real terms this means that a distance just short of half a kilometre would be travelled every second!

Extension

A similar but more difficult scenario would be to determine the relative speed for a building (or any other object) in New Orleans (U.S.A.) that is located at 30° North parallel of latitude.

To determine the rotational speed for a point that is at latitude of 30° N it is necessary to modify the model being used. It would seem plausible to assume that since a point is at a higher latitude that if a ‘slice’ of the Earth is taken to reveal a plane surface then this would be a circle with a smaller diameter. A rotation in a day would travel a lesser distance and so the speed of rotation would be expected to be slower.

Now we need to consider the shape made, and angles formed when the Earth is viewed showing the North and South Poles. In the diagram below the North Pole (N), South Pole (S), the latitude of 30° N showing building located in New Orleans (B), the centre of the Earth (O) and the diameter of the Earth (d) as well as the radius of the Earth (r) at the given latitude. This is shown in the following diagram;



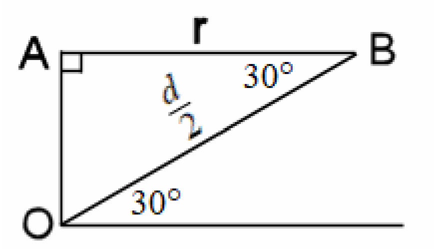
It is necessary to work out the radius of the Earth at the point which is at the latitude of 30°N. Following this it is important to determine the distance travelled by a point on the Earth’s surface at the latitude of 30° N. From this point it is possible to determine the speed of rotation at the desired latitude.

The plane through the centre of the Earth at the Equator is a circle while the plane through the Earth at any other latitude is parallel to the larger and has a smaller diameter / radius.

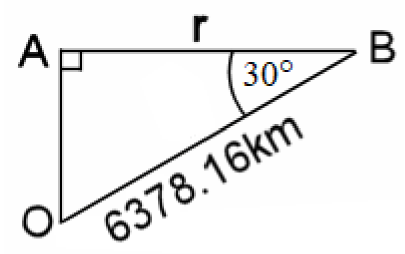
It is assumed that every point on the surface of the Earth is equidistant from the centre of the Earth, meaning that the radius is equal. This is not entirely correct but for calculation purposes and for the scale of the Earth it is easier to assume.

In the diagram above, d is the diameter of the Earth while OB is half the diameter (or radius) of the Earth.

We can now make a couple of diagrams that will model the measurements being used.



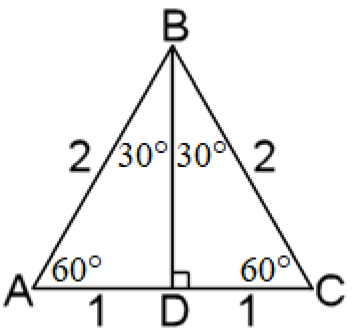
Using the diameter as 12,756.32km, this means that the radius of the Earth is 6378.16km and this in turn gives the model as;

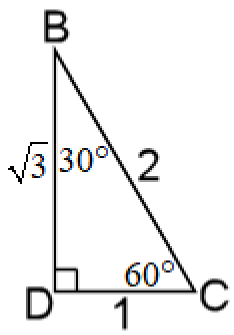


It can now be seen that triangle OAB is right angled. From the angle (B), the sides AB (adjacent to angle B) and OB (hypotenuse opposite the right angle) can be used with the cosine trigonometric ratio, giving;

$$\cos 30° = \frac{r}{6378.16}$$
$$r = 6378.16 \times \cos 30°$$
$$r = 6378.16 \times \frac{\sqrt{3}}{2}$$
$$r = 5523.648589$$
$$r \approx 5523.65$$

<u>Note</u>:
The exact value for $\cos 30°$ is used and this is found from the equilateral triangle shown below, where the length of each equal side is given as 2 units. Consider the equilateral triangle ABC;





In triangle ABC, the perpendicular bisector of AC passes through D, bisecting angle ABC.

By Pythagoras' Theorem, with hypotenuse of 2 units, one shorter side of 1 unit, the length of the other shorter side is $\sqrt{3}$ units.

Using the radius of the Earth at the Parallel of Latitude of 30°N upon which New Orleans is located it is now possible to determine the distance travelled in one day by a single rotation of the Earth (Circumference at this latitude);

$$
\begin{aligned}
C &= \pi d \\
C &= 2\pi r \\
&= 2 \times \pi \times 5523.65 \\
&= 34706.11652 \\
&\approx 34706\text{km}
\end{aligned}
$$

Using the circumference at the latitude of 30°N, this is the distance of rotation of the Earth at this position during a day. From this, we can calculate the speed of rotation at this latitude;

Speed (expressed in km/h)

$$
\begin{aligned}
S &= \frac{D}{T} \\
&= \frac{34706km}{23.93h} \\
&= 1450.313414 \\
&\approx 1450\text{km/h}
\end{aligned}
$$

This means that any object on the surface of the Earth lying at a latitude of 30°N will be travelling in space at a speed of 1,450 km/h.

Speed Conversion (from km/h to metres per second - m/s)

It is important to know the following information in order to convert from one unit of speed (km/h) into another unit of speed (m/s).

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\begin{aligned}
1h &= 60\min \\
&= 60 \times 60\sec \\
&= 3600\sec
\end{aligned}
$$

$$
\begin{aligned}
S &= \frac{D}{T} \\
&= \frac{34706 \times 1000\text{m}}{23.93 \times 60 \times 60\text{s}} \\
&= 402.8648373 \\
&\approx 403\text{m/s}
\end{aligned}
$$

Hence, any object in New Orleans at latitude 30°N would be travelling at about 403 m/s. This would be approximately 62m/s slower than at the Equator.